transformation the $x_i(\alpha,\beta)$ be the same, then corresponding to those orders is to be inferred the existence of the derivatives

$$D_\alpha D_\beta x_i = \sum_j \xi_{ij} D_\beta x_j = \sum_j \eta_j \xi_{ij} = Y\xi_i,$$

$$D_\beta D_\alpha x_i = \sum_j \eta_{ij} D_\alpha x_j = \sum_j \xi_j \eta_{ij} = X\eta_i.$$

If these are to agree the commutator must vanish at all points actually occurring in the transformation; a similar limitation in the meaning of "identical" may be understood in the proof of sufficiency.

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## *THE INTENSITIES OF X-RAYS OF THE L SERIES*
## *II. THE CRITICAL POTENTIALS OF THE PLATINUM LINES*

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*Introduction.*—This work is a continuation of that of Webster and Clark, reported in these PROCEEDINGS[1] in 1917. Part of the present work was done at Harvard University with the apparatus described in the earlier paper, and part with my new apparatus at the Massachusetts Institute of Technology. The object in view is the investigation of the laws relating intensity to potential, for the L-series lines, for the purpose of comparison with current theories of X-ray spectra, and the present paper deals with the determination of the critical potentials of the platinum lines of medium intensity, the stronger ones having been reported in the previous paper and the fainter ones, observed only in tungsten by Dershem[2] and Overn,[3] being so faint as to require a much more prolonged study.

*Apparatus.*—The work at Harvard showed that with the slit widths needed for accurate intensity measurements it would be difficult to work with certainty on any lines but the strong ones, when the voltage was near the critical value. As the previous work of the author on the rhodium K series[4] had shown that photography gave good results in such work it was decided to use it here. In this case, where the lines are many and scattered and faint, the best instrument seemed to be the bent mica spectrograph of de Broglie and Lindemann[5] with which they have obtained excellent spectrograms. These show spectra of six orders, called first to sixth inclusive, though I think the last one, from its angle, must really be the seventh rather than the sixth, which must be very faint. The grating space is about 10 Ångstroms and the third and fifth orders are the strongest.

In the present work, to increase the intensity even at a sacrifice of resolving power, I used a long radius, either 40 or 80 cm., with the tube 75 cm. from the mica and the plate as shown in figure 1. In practice, even with the best mica, there are faults in its structure that make it necessary to take several photographs at each voltage with different positions of the mica so as to insure clear reflection for each line in one case or another. To avoid scattered rays the mica was held on the edges of two

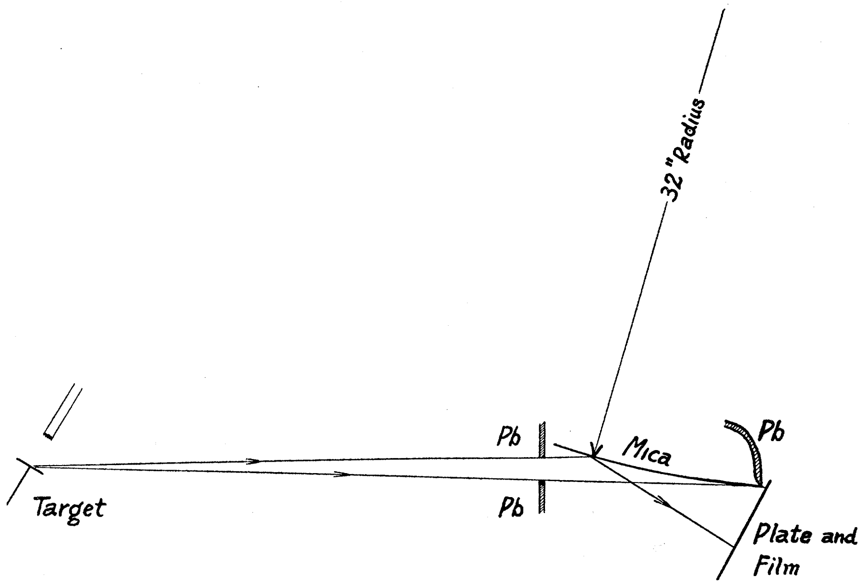


FIG. 1

brass plates, far enough apart so that no direct rays struck them, and the direct beam through the mica was caught in a sheet of lead. The lines were identified from a chart made from the wave-length measurements of Siegbahn and Friman,[6] and the voltmeter (Chaffee type, as in the previous work) was calibrated by the wave-length $\lambda_{min}$, of the high frequency end of the third order spectrum as determined from this chart. In all exposures near any critical voltages the voltage was regulated to a mean deviation of $^1/_5$ to $^1/_3$ of 1% from the average value.

An important improvement in the voltmeter, introduced in the work at Harvard, was the use of a single insulator only, directly under the suspended system, with fairly good electrostatic screening. This prevents disturbances from indeterminate charges on the insulator which may otherwise make serious errors.

Each spectrogram was taken on one or two Eastman Dupli-tized X-ray films backed by a Seed Graflex plate. For locating very faint lines this combination was useful because with the plate and film, or two films, in contact after development any relative motion of the two would make

the spectrum flash out stronger when the images coincided. This method worked especially well when they were still wet. The parts where the general radiation was not too strong were also examined by holding the film against a white paper in a strong light, and a negative lens was also used. Since these methods were needed it is obviously impossible to show the faint lines in a reproduction even when there was no doubt of their existence in the original. Therefore, no reproduction is attempted here.

I wish here to express my thanks to Mr. D. S. Piston, who took most of the photographs in this investigation.

The potential was furnished by a modified form of Hull's[7] apparatus shown in figure 2. The 500-cycle current came from a Crocker-Wheeler motor generator loaned by the Submarine Signal Co., through the generosity of Mr. H. W. J. Fay. The transformers and inductances were made mostly from parts of induction coils contributed by the Boston City Hospital, through the kindness of Dr. F. H. Williams. The 60-cycle one had the small secondaries so placed that the heating current in either kenotron was the same whether the other was on or not. The condensers were of glass plates and sheet iron, immersed in oil and protected as in Hull's apparatus, by corona gaps.

With connection $A$, shown by the short-dash line, only one kenotron is used, and the potential on it rises to more than twice that of the D. C. line, while the frequency of the fluctuation on that line is 500~, the same as that of the transformer. But with connection $B$, using two kenotrons, the potential on each is that of the D. C. line itself, that of the transformer is half that amount, and the fluctuation frequency on the D. C. line is doubled. It should be noticed that to produce this double frequency it is most essential that no part of the transformer secondary or first or second condensers should be grounded.

The constants of the apparatus were approximately as follows: $C_1 = 3 \times 10^{-3}$ mfd, $C_2 = C_3 = 1.5 \times 10^{-3}$ mfd, $L_1$ (sum of the two sides plus twice the mutual inductance) $=$ 20,000 henries, $L_2$ $=$ 15,000 henries. Such large inductances are not strictly necessary, and were used only because they were at hand. They were measured by using 90 kv. (measured by spark) at 500~ with an A. C. voltmeter (current type) in series in the middle of the line. As the voltage fluctuation is approximately $\frac{I}{C_1C_2C_3L_1L_2\omega^5}$, it is extremely small, probably not over a few volts under the worst conditions. The resistance of $L_1$ and $L_2$ together, measured by D. C., was about 0.3 megohm. When running, a current type voltmeter in series with $C_2$ and $C_3$ showed no measurable current, though of course it showed easily the current in $C_1$.

Incidentally, it may be noted that the above expression for the fluctuation should contain also the factor $(1 - C_2M\omega^2)$, where $M$ is the mutual inductance between $L_1$ and $L_2$. This factor could readily be used to neu-

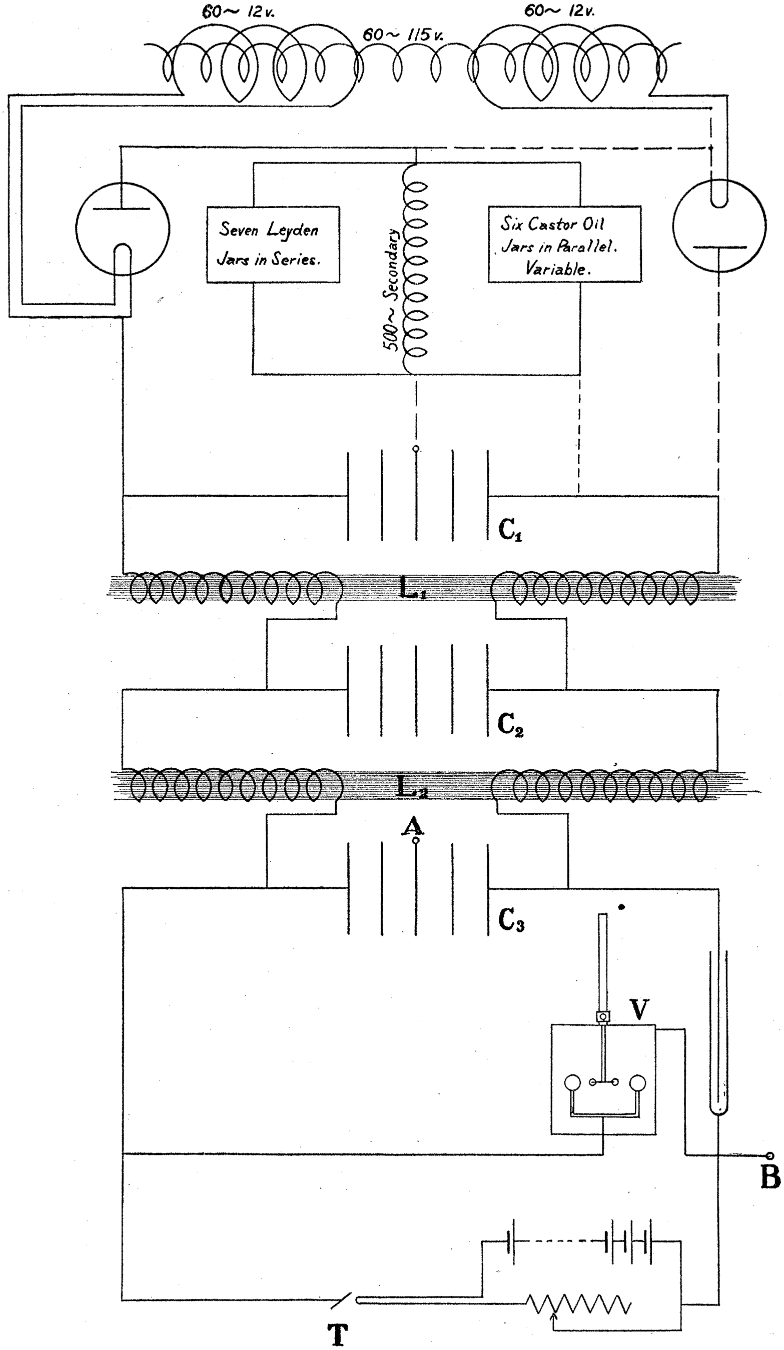


FIG. 2

Note: Short-dash line, connection *A* only. Long-dash lines, connection *B* only. Ground only at point *A* at high potentials and only at point *B* at low.

tralize the slight remaining fluctuation, as at 2 meters distance, with $L_1$ and $L_2$ parallel, $M = 100$ henries. In this case, however, no attempt was made to adjust $M$ accurately, but $L_2$ was tipped to a nearly vertical position, 2 meters from $L_1$, which was horizontal, and $L_1$ was set perpendicular to the 500 $\sim$ transformer, and also to the 60 $\sim$ one, which were both horizontal and far apart.

An important point is the condensers across the secondary of the transformer. To eliminate unsteadiness due to fluctuations on the 115-volt D. C. line, that ran the motor-generator, the generator field was excited by a storage battery and the variable condenser was then adjusted for resonance. Under these conditions, the potential on $C_1$ being a maximum with respect to speed changes, a small change in the speed of the motor could affect it only by a second order amount, always negative, therefore, not such as to bring out spectrum lines falsely. This arrangement proved invaluable in making long exposures. The only fluctuations that ever made trouble were small ones due to current changes produced by the traces of gas that cannot be removed from the tube without melting the platinum target to do so.

I wish here to express my thanks to Mr. R. M. Frye for valuable assistance in the construction of this apparatus.

The X-ray tube was the same one used in the previous work, but, through the kindness of Dr. Coolidge, it had been provided with a thin window of a type used by Blake and Duane,[8] made by blowing a thin, spherical shell, convex inward. This greatly reduced the exposures required.

*Results.*—Considering each line separately, the results are as follows:

$\alpha_1$. This line was definitely shown by the ionization method to belong to $L_1$, and fixed the critical potential of $L_1$ most accurately, the average of the best measurements being $11.47 \pm 0.05$ kv. Since conditions were adapted for intensity measurements but no attempt was made to measure wave-lengths, the value of $h$ found there will not be used, but rather the value recommended by Birge,[9] which is $6.554 \times 10^{-27}$ erg sec., with Millikan's $e$. This gives $\lambda_{L_1} = 1.076 \pm 0.005$ Å. The films show $\alpha_1$ very strong when $\lambda_{min} = 1.01$ Å, and not at all at 1.08.

$\alpha_2$. In the mica films this line is resolved from $\alpha_1$ only in the 5th order, but as this order is strong, it shows clearly when $\lambda_{min}$ is around 0.90 Å, and definitely, though faintly, at 1.01. Since the critical wave-length of $L_2$ is 0.935 Å (see $\beta_1$), this places $\alpha_2$ in $L_1$. The ratio of its intensity to that of $\alpha_1$ was tested at Harvard by the ionization method, with results shown in figure 3. Since the ratio is the same at 40.0 kv. as at 17.85, the law of constant intensity ratios for lines of the same series holds in this case.

*Ir* $\alpha_1$. This line, due to an impurity of iridium in the target, shows faintly but definitely when $\lambda_{min} = 1.01$ Å, but not at 1.08.

$l$. This line is too faint to show near its critical potential in any order

but the third, and there it is covered at high voltages by the fourth order $\beta_1$. But $\beta_1$ belongs to $L_2$, and even its strong third order line is gone at $\lambda_{min}$ = 0.945 Å, while a line still appears at the position of 4 $\beta_1$ and 3 $l$. This must be 3 $l$. It is definitely visible though somewhat fainter than

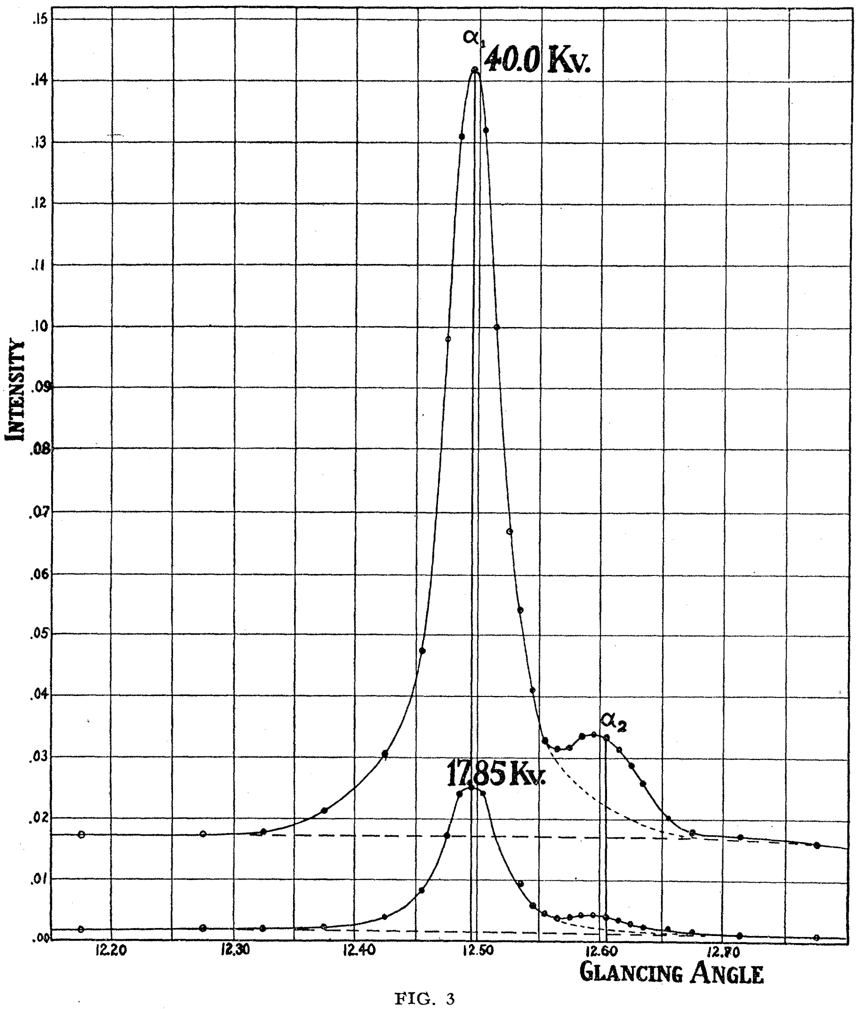


FIG. 3

Reckoning the intensities by measurement of the vertical lines drawn to the peaks in the figure, the ratio of $\alpha_1$ to $\alpha_2$ is 1.11 at 17.85 kv. and 1.10 at 4.00.

3 $Ir$ $\alpha_1$. It is doubtful at 1.01 Å, but the fact that it shows at 0.945 places it in $L_1$.

$\beta_2$. This line shows strongly when $\lambda_{min}$ = 1.01 Å, but not at all at 1.08, as one might expect from the ionization work previously reported, which placed it in $L_1$.

$\beta_5$. This line shows well at high voltages, and faintly but distinctly when $\lambda_{min} = 1.01$ Å. This places it in $L_1$.

$\beta_6$. In the calcite spectrograms taken at Harvard there is a faint companion of $\beta_5$ at a wave-length greater by $0.010 \pm 0.002$ Å. This also shows in some high resolution mica films, where the tube was 3 meters from the mica instead of 75 cm., and in a few of the low resolution mica films, at $\lambda_{min} = 0.88$, 0.910 and 0.945 Å. In all the others $\beta_5$ is wide. The fact that it shows as low as 0.945 Å places it in $L_1$, and its intensities in the calcite films indicate that it follows qualitatively the constant intensity ratio law. At least it does not violate it in any such way as an $L_2$ line would when compared with an $L_1$. Since $\beta_5$ and $\beta_6$ are both in $L_1$ their wave-length difference would be nearly the same in tungsten as in platinum. $\beta_6$ is, therefore, probably the analogue of the tungsten line discovered by Dershem and given by Overn as 1.2212 Å, $\beta_5$ being his line 1.2097 Å.

$\beta_1$. This line belongs to $L_2$ and gave the most definite results by ionization for determining $\lambda_{L_2}$. With the present $h$ this is $0.935 \pm 0.004$ Å. The films show $\beta_1$ plainly at 0.910 Å, but not at all at 0.945 Å even in the third order.

$\eta$. The third order of this line unfortunately coincides with the fourth of $\gamma_2$, which belongs, as noted below, to $L_2$. But the fourth order is faint and the third strong, and the fourth $\gamma_1$ which is very near $3\ \eta$ and $4\ \gamma_2$ is but very little stronger than they. Since $3\ \gamma_1$ is much stronger than $3\ \gamma_2$ this means that most of the intensity of this combined line is that of $3\ \eta$. Now the combined line and $4\ \gamma_1$ show the same intensity ratio, as nearly as one can tell, at all voltages, and are visible down to $\lambda_{min} = 0.910$ Å, but not at 0.945. At these low voltages $4\ \gamma_2$ alone would certainly be invisible, and if $\eta$ and $\gamma_1$ were not in the same series the change in their ratio would be unmistakable even with no measuring instruments. Hence, we may fairly confidently assign $\eta$ to $L_2$.

$\beta_4$. This line gave results by ionization showing a critical wave-length $0.935 \pm 0.02$ Å, the accuracy not being equal to that obtained with $\beta_1$ because it is rather faint. The films, however, show a faint line in this position even at $\lambda_{min} = 1.01$ Å, far below the critical potential of $L_2$. This is probably the same as a line which is just resolved from *Pt* $\beta_4$ in the calcite and high resolution mica spectrograms. It is called *Ir* $\beta_2$ because its wave-length is that of the iridium line and it obviously does not belong to $L_2$. But as all lines are rather wider in the mica photographs and the work of Dershem and Overn showed a line in tungsten that might be near this point in platinum, further work with a high resolving power and long exposures is needed. As the present spectrograph is unsuitable for this and several other problems presented by the very faint lines, a more suitable one is now being constructed by Mr. F. C. Hoyt.

*Ir* $\beta_1$ *and Ir* $\beta_2$. These lines appear in the calcite and high resolution

mica films, and behave like $Pt\ \beta_1$ and $Pt\ \beta_2$, the changes in relative intensities with voltage being very striking in both cases.

$\beta_3$. This line is obscured by $\beta_2$ except in the calcite films. Its behavior there is similar to that of the $L_2$ lines and quite different from that of the $L_1$. It must be in either $L_2$ or $L_3$.

$\lambda_1$. The films show this line clearly when $\lambda_{min} = 0.910$, but not at all at 0.945, confirming the ionization work that assigned it to $L_2$.

$\gamma_2$ *and* $\gamma_3$. These two lines are not resolved except in three films at 35, 25 and 18 kv. In these the tube was removed 3 meters from the mica instead of 75 cm. In this case the exposure at 35 kv. was 7 hours and at 18 kv., 30 hours. In these films $\gamma_2$ and $\gamma_3$ are just resolved in the third order and well separated in the fifth. These films were taken for three purposes, to confirm the existence of $\beta_6$, $Ir\ \beta_1$ and $Ir\ \beta_2$, to get the absorption spectrum discussed below, and to see if any difference could be found in the intensity ratio of $\gamma_2$ and $\gamma_3$ at different voltages. $\gamma_3$ in each film is fainter than $\gamma_2$, and it appeared slightly more so at 18 kv. than at 35, but the difference in the ratio was too small to base any definite conclusions on it, though if it exists they must belong to different series. The low resolution films show the combined line $\gamma_{2,3}$ well at high potentials, faintly but definitely at $\lambda_{min} = 0.895$ Å, and it is barely visible under the best conditions at 0.907. Since $\lambda_{\gamma_4} = 0.900$ Å, this line cannot be in the same series with $\gamma_4$ but must have a critical wave-length longer than 0.907 and, therefore, presumably belong to $L_2$ where $\lambda_{L_2} = 0.935$ Å. Now the absorption spectra discussed below indicate that $\gamma_3$ is distinctly more absorbed than $\gamma_2$, showing an absorption limit, $A_2$, corresponding to $L_2$, lying between them. This apparently means that $\gamma_3$ cannot belong to $L_2$ and, therefore, that $\gamma_2$ is probably the line showing at $\lambda_{min} = 0.907$ Å. But it does *not* mean definitely that $\gamma_3$ has the same critical potential as $\gamma_4$, although the natural assumption is that it has. This is important in connection with Sommerfeld's theory to be discussed in the next paper.

$\gamma_4$. This line is so near the bromine absorption limit that any work on it is uncertain, and with the mica spectrograph 3 $\gamma_4$ is obscured by 2 $Ir\ \alpha_1$ and 5 $\gamma_4$ by 4 $\beta_4$. But since $\lambda_{\gamma_4} < \lambda_{L_2}$ or $\lambda_{L_1}$ it can hardly belong to either of these series, and must be ascribed to $L_3$.

*The Faint Lines.*—In this class are included the lines found only by Dershem and Overn, in tungsten. For these, further work will be necessary, with high resolving power and very long exposures.

*The Critical Points.*—The question arises whether the critical wave-lengths determined by potentials are identical with the absorption limits, as they seem to be in the K series. The critical wave-lengths found by Clark and me for $L_1$ and $L_2$ were identical, within limits of error, with the wave-lengths of $\beta_5$ and $\gamma_2$, respectively, provided $h = 6.554 \times 10^{-27}$ erg sec., and, therefore the results of various measurements of absorption limits are compared with these lines in the following table:

TABLE 1

| ELEMENT | $\lambda_{A_1}$ | $\lambda_{\beta_5}$ | $\lambda_{\beta_5}-\lambda_{A_1}$ | $\lambda_{A_2}$ | $\lambda_{\gamma_2}$ | $\lambda_{\gamma_2}-\lambda_{A_2}$ | $\lambda_{A_3}$ | $\lambda_{\gamma_4}$ | $\lambda_{\gamma_4}-\lambda_{A_3}$ |
|---|---|---|---|---|---|---|---|---|---|
| Tungsten | 1.230[10] | 1.2097[3] | —0.020 | 1.081[10] | 1.0596[3] | —0.021 | 1.025[10] | 1.0263[3] | +0.001 |
| Tungsten | 1.215[11] | 1.2097[3] | —0.005 | | | | | | |
| Platinum | 1.070[11] | 1.072[6] | +0.002 | | | | | | |
| Platinum | 1.072[12] | 1.072[6] | 0.000 | 0.934[12] | 0.033[6] | —0.001 | | | |
| Gold | 1.041[11] | 1.035[6] | —0.006 | 0.901[11] | 0.898[6] | —0.003 | 0.861[11] | 0.869[6] | +0.008 |
| Gold | 1.042[12] | 1.035[6] | —0.007 | 0.914[12] | 0.898[6] | —0.016 | | | |
| Mercury | 1.009[11] | 1.013[13] | +0.004 | | | | | | |
| Thallium | 0.977[11] | 0.977[6] | 0.000 | 0.843[11] | 0.844$_6$ | +0.001 | | | |
| Lead | 0.948[11] | 0.953[13] | +0.005 | 0.814[11] | 0.820[6] | +0.006 | | | |
| Lead | 0.949[14] | 0.953[13] | +0.004 | 0.813[14] | 0.820[6] | +0.007 | 0.781[14] | 0.792[6] | +0.011 |
| Bismuth | 0.924[11] | 0.923[6] | —0.001 | 0.789[11] | 0.794[6] | +0.005 | 0.756[11] | 0.762[6] | +0.006 |
| Thorium | 0.760[11] | 0.767[13] | +0.007 | 0.627[11] | 0.637[13] | +0.010 | 0.607[11] | 0.610[13] | +0.003 |
| Uranium | 0.721[11] | 0.726[13] | +0.005 | 0.591[11] | 0.598[13] | +0.007 | 0.567[11] | 0.571[13] | +0.004 |
| Mean.................... | | | —0.001 | | | 0.000 | | | +0.006 |

From this table it would appear that $A_1$, $A_2$ and $A_3$ are probably at nearly the same wave-lengths as $\beta_5$, $\gamma_2$ and $\gamma_4$, respectively, in every element. But most of these results were obtained with different apparatus for the two kinds of measurement, and there is considerable disagreement. As even a slight difference in wave-length one way or the other is of great theoretical interest, it seemed desirable to check this point by using a screen of $H_2PtCl_6$ on blank films. This was attempted in two of the high resolving power spectrograms. The results were not very satisfactory, owing to the strong absorption by the other substances than platinum in the screen, and the presence of the less absorbed radiation of higher frequencies, in the fifth order, at the most interesting part of the third. But they seem to indicate that $A_1$ is within a very few thousandths of an Ångstrom of $\beta_5$, and that $A_2$ is between $\gamma_2$ and $\gamma_3$, $\gamma_3$ being apparently the more strongly absorbed of these lines. This is consistent with the conclusion reached above, that $\gamma_2$ belongs to $L_2$ and $\gamma_3$ to $L_3$, but the evidence is unsatisfactory and none was obtained on $A_3$. This apparatus was not designed for high resolving power but it is hoped that this point will be settled with the other apparatus now being constructed by Mr. F. C. Hoyt for work on the fainter lines.

*Summary of Results.*—We have assigned the lines to the various series as follows:

To $L_1$, $l$, $\alpha_2$, $\alpha_1$, $\beta_2$, $\beta_6$ and $\beta_5$;

To $L_2$, $\eta$, $\beta_4$, $\beta_1$, $\gamma_1$, $\gamma_2$ and perhaps $\beta_3$;

To $L_3$, $\gamma_4$, probably $\gamma_3$ and perhaps $\beta_3$.

Unassigned, all of Dershem's and Overn's faint lines except $\beta_6$, which is probably one of them.

Critical points:
$\lambda_{L_1} = 1.076 \pm 0.005$ Å, approximately $= \lambda_{\beta_5}$;
$\lambda_{L_2} = 0.935 \pm 0.004$ Å, approximately $= \lambda_{\gamma_2}$;
$\lambda_{L_1}$, $\lambda_{L_2}$ and $\lambda_{L_3}$ are probably exactly equal to $\lambda_{A_1}$, $\lambda_{A_2}$ and $\lambda_{A_3}$ though more work is needed on this point.

Intensity ratios: constant for any pair of lines of the same series, with changes in voltage, as far as they have been investigated.

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# *THE EFFECT OF PHYSICAL AGENTS ON THE RESISTANCE OF MICE TO CANCER*

BY JAMES B. MURPHY

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH, NEW YORK CITY

Read before the Academy, November 10, 1919

The report which I wish to present today has to do with further progress in the work which I reported before the Academy in June, 1915. I will review that work briefly in order to orient you with the observations made since then.

The fundamental point in immunity to transplanted cancer is that there are two types of resistance, the so-called natural and induced immunity. Mice may be rendered resistant by an injection of a quantity of homologous living tissue given at least a week or ten days before the cancer inoculation; this is called induced immunity. A variable proportion of mice inoculated with a transplantable tumor will be resistant; this is called natural immunity. The histological manifestation of resistance about an introduced cancer graft in these two types of immunity is the same and